THERMAL CONDUCTIVITY AND THERMAL DIFFUSIVITY IN GRAIN LAYERS OF SOME AGRICULTURAL PRODUCES

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A method of analysis is shown which is based on the principle of a constant heat flux. The results of measurements made on broad bean, bean, pea, corn, rape, and lupine seeds are given. These results are presented in a general form.

Heat conduction in a layer of moist broad bean, bean, pea, corn, and rape was studied with an apparatus shown schematically in Fig. 1. A transient mode of heat conduction was maintained here as that in a semi-infinite body thermally insulated along the sides and heated at one end. Under such conditions, and also after the carefully mixed grain had been placed inside the apparatus, the equation of heat conduction

$$\frac{\partial t(x, \tau)}{\partial \tau} = a \frac{\partial^2 t(x, \tau)}{\partial x^2} \tag{1}$$

was defined inside the body

$$0 \leqslant x < +\infty$$

for $\tau \geq 0$ with the initial condition:

$$t(x, 0) = t_0 = \text{const} \tag{2}$$

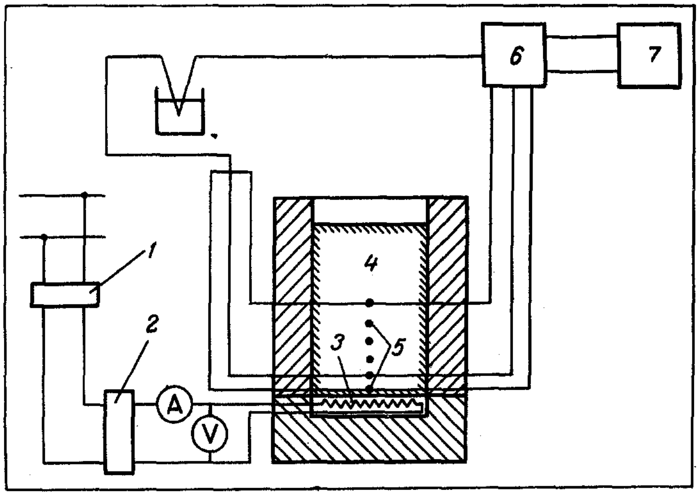

Fig. 1. Schematic diagram of the test apparatus: 1) voltage stabilizer; 2) autotransformer; 3) heater; 4) measurement chamber; 5) thermocouples; 6) switch; 7) galvanometer.

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TABLE 1. Solution to Eq. (1)

Secondary boundary condition	Equation	Solution	Equation
$t(0, \tau) = t_1$	(4)	$\frac{t(x, \tau) - t_0}{t_1 - t_0} = \operatorname{erf} \frac{x}{2\sqrt{a\tau}}$	(a)
$t(0, \tau) = k\tau$	(5)	$t(x, \tau) - t_0 = 4k\tau\, i^2 \operatorname{erfc} \frac{x}{2\sqrt{a\tau}}$	(b)
$t(0, \tau) = k\sqrt{\tau}$	(6)	$t(x, \tau) - t_0 = k\sqrt{\pi\tau}\, i \operatorname{erfc} \frac{x}{2\sqrt{a\tau}}$	(c)

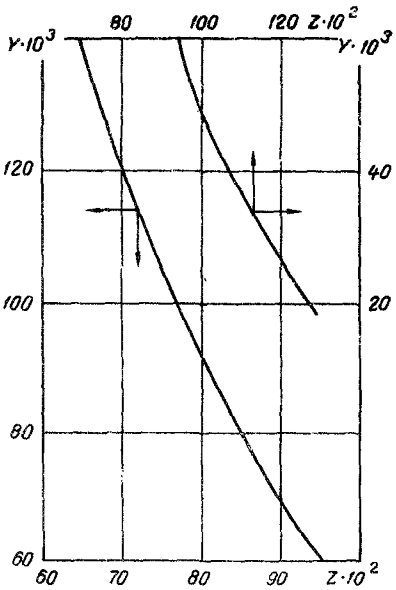

Fig. 2. Graph of the function Y i·erfcZ.

and with the first boundary condition:

$$t(+\infty, \tau) = t_0 = \text{const}. \tag{3}$$

The approximate solution to Eq. (1) is shown in Table 1 for the following second boundary conditions, of possible practical significance:

$$t(0, \tau) = t_1 = \text{const}, \tag{4}$$

$$t(0, \tau) = k\tau, \tag{5}$$

$$t(0, \tau) = k\sqrt{\tau}. \tag{6}$$

From the practical point of view, it is worthwhile to perform tests under the conditions defined by Eqs. (4) and (6).

The boundary condition defined by Eq. (4) is realizable in practice by passing a stream of liquid at a constant temperature over the heater surface. It is impossible to attain the desired constant temperature at the heater surface immediately. This temperature will be reached only after some time and, in practical terms, the boundary condition thus becomes distorted. The error arising as a consequence depends on the heater construction and on the temperature at which it is desirable to measure the thermal conductivity. The boundary condition defined by Eq. (6) is realizable in practice by maintaining the electric-heater voltage and current constant. Under these conditions we obtain a constant heat flux from the heater surface. Measurements of the heater surface temperature make it possible to determine the coefficient in Eq. (6) with an accuracy which will make this method suitable for thermal-conductivity meansurements.

The thermal conductivity was measured with the initial condition as defined by Eq. (2) and with the boundary conditions as defined by Eqs. (3) and (6).

From Eq. (a) (Table 1) and the Fourier number Fo_x we obtain an equation which defines the thermal conductivity of a grain layer:

$$\lambda = \frac{c\gamma x^2 Fo_x}{\tau}. \tag{7}$$

In order to perform calculations with this equation of thermal conductivity for a grain layer, the specific heat of the seeds in the special apparatus was varied as a function of the moisture content. This yielded the following equation

$$c = c_0 + 4186.8\, u, \tag{8}$$

valid within the tested range of moisture content not exceeding 0.4 kg water per 1 kg of dry mass. Also the specific and the layer bulk densities of the various seeds were measured. It had been established, as a result, that the specific mass of seeds within the tested range of moisture content was independent of the latter. The bulk density, however, varied together with the moisture content in the seeds.

TABLE 2. Results of Temperature Measurements in a Grain Layer

Time of measurement	Ice	Distance from the heater						Medium
		0	20	30	40	60	80	
	division on the galvanometer scale							
0	0		59	52	52	52	52	52
30	0	185	81	64	57	53	52	52
35	0	197	89	69	60	55	53	52
40	0	205	95	75	63	55	53	52
45	0	215	102	78	65	57	53	52
50	0	223	110	83	69	55	54	53
55	0	228	115	87	71	59	55	52
65	0	235	121	92	75	61	56	53
70	0	242	128	98	80	64	58	53
75	0	246	132	102	82	65	57	53
80	0	249	137	105	85	67	58	52
85	0	255	142	111	90	70	60	53
90	0	256	145	113	92	71	60	52

TABLE 3. Thermal Conductivity of Broad Bean, Bean, Pea, Corn, Lupine, and Rape Seed Layers along with Some Other Parameters Referred to Seeds with a Zero Moisture Content

Seeds	Thermal conductivity, W/m·°C	Specific heat, J/kg °C	Density, kg/m³		Mass of 1000 grains, g	Equivalent grain diameter, m	Porosity %
			layer bulk	specific mass			
Broad bean	0,140 ± 0,001	1344	763	1305	440,0	0,00864	41,5
Bean	0,136 ± 0,001	1210	808	1316	177,6	0,00637	38,6
Pea	0,129 ± 0,001	1231	774	1321	171,6	0,00628	41,4
Corn	0,158 ± 0,002	1679	708	1212	301,1	0,00780	41,6
Lupine	0,123 ± 0,002	1285	753	1247	118,5	0,00566	39,6
Rape	0,160 ± 0,001	2244	553	1003	4,9	0,00080	44,9

In order to determine the Fourier number Fo_x from Eq. (b) (Table 1), the following expression

$$\frac{t(x,\tau)-t_0}{k\sqrt{\pi\tau}} = i\,\mathrm{erfc}\frac{1}{2\sqrt{Fo_x}} \tag{9}$$

was derived and plotted on a graph (Fig. 2) with the aid of the table of function values for

$$Y = i\,\mathrm{erfc}\,Z. \tag{10}$$

On the basis of Fig. 2, for any instant $t(x, \tau)$ of time during the test, we could find the corresponding value of Fo_x. As had been mentioned already, the value of parameter k in Eq. (9) was determined by numerous measurements of the heater surface temperature as a function of the heating time.

The measurement errors were estimated considering the measured quantities to be mutually independent. This conforms to reality, since the error in measuring the time is minimal. It follows from Eq. (7) that the relative error in the thermal conductivity measurement which is due to systematic errors can be estimated from the equation:

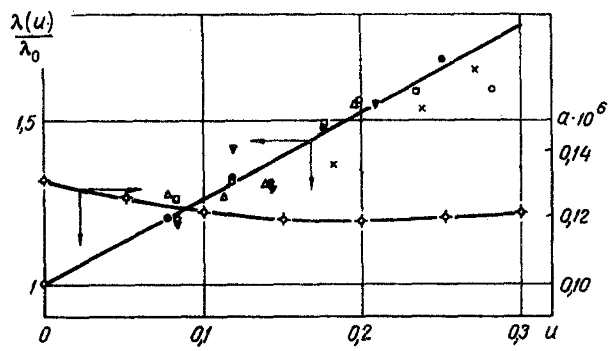

Fig. 3. Thermal conductivity and thermal diffusivity (m^2/sec) in a layer of moist grain.

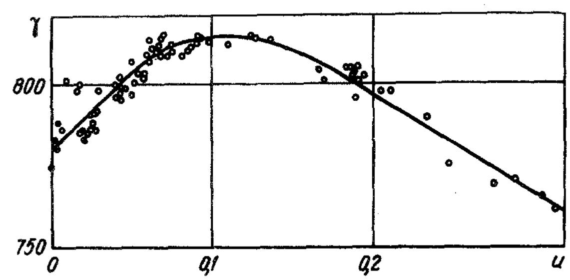

Fig. 4. Bulk density (kg/m^3) of a pea layer as a function of the moisture content (kg/kg) in the peas of the layer.

$$\left|\frac{\Delta\lambda}{\lambda}\right| = \left|\frac{\Delta c}{c}\right| + \left|\frac{\Delta\gamma}{\gamma}\right| + \left|\frac{\Delta\tau}{\tau}\right| + 2\left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta \mathrm{Fo}_x}{\mathrm{Fo}_x}\right|, \tag{11}$$

where, according to (9) and (10), the procedure for estimating the error in the Fourier number is:

1) assuming $t(x, \tau) - t_0 = t$ we have

$$\left|\frac{\Delta Y}{Y}\right| = \left|\frac{\Delta t}{t}\right| + \frac{1}{2}\left|\frac{\Delta\tau}{\tau}\right| + \left|\frac{\Delta k}{k}\right|; \tag{12}$$

2) on the basis of this equation, the error ΔZ is found from Fig. 2;

3) the error in the Fourier number is then determined from the relation:

$$\left|\frac{\Delta \mathrm{Fo}_x}{\mathrm{Fo}_x}\right| = 2\left|\frac{\Delta Z}{Z}\right|. \tag{13}$$

After the values of the individual terms in Eqs. (11), (12), and (13) have been determined, the range (x, τ) is then determined within which the values of $t(x, \tau)$ make it possible to calculate the thermal conductivity in a grain layer so that under given conditions the error of a single measurement, due to the total of systematic errors, is minimal. Such a range is shown, for illustration, in Table 2.

The results of thermal conductivity measurements in a layer of perfectly dry grain, with a zero moisture content, are shown in Table 3 along with some physical parameters.

The graph in Fig. 3 represents the thermal conductivity in a grain layer referred to the thermal conductivity of a layer with perfectly dry grain. From this graph follows the equation:

$$\frac{\lambda(u)}{\lambda_0} = 1 + 2.668\,u, \tag{14}$$

which has been established by the method of least squares with a 0.97 correlation coefficient. This equation allows one to find the thermal conductivity of a layer of a given grain species within the range of moisture content

$$0 \leqslant u \leqslant 0.4 \text{ kg/kg}, \tag{15}$$

if the thermal conductivity is known for a layer of that grain when perfectly dry.

From the thermal conductivity in a layer of moist grain one can determine the thermal diffusivity in a layer of moist grain:

$$a(u) = \frac{\lambda(u)}{c(u)\,\gamma(u)}. \tag{16}$$

Using Eq. (14) and also Eq. (8) in the form:

$$c(u) = c_0 + 4186.8\,u, \tag{17}$$

we obtain the following equation for the thermal diffusivity in a layer of moist grain:

$$a(u)=\frac{\lambda_0(1+2{,}668\ u)}{(c_0+4186.8\,u)\gamma(u)}. \tag{18}$$

It has not yet been established mathematically how the layer bulk density of grains in a test setup depends on the moisture content in the grain, but calculations can be performed using the experimental data shown here graphically. For pea, the thermal conductivity of which has been determined, the layer bulk density as a function of the moisture content in the seed is shown in Fig. 4. This relation is of the same kind for other seeds as well. The thermal conductivity of a pea layer (Fig. 3) has been determined with the aid of Eq. (18), Fig. 4, and the data in Table 3. Even with the relation between the bulk density of a grain layer and the moisture content in the layer represented by an empirical equation only, it becomes possible to express the diffusivity in a layer of moist grain mathematically as a function of the moisture content in the grain.

NOTATION

t is the temperature;
x is the distance from the heater surface;
τ is the time;
a is the thermal diffusivity;
k is the temperature rise coefficient, at the heater surface;
λ is the thermal conductivity of a grain layer;
$\lambda(u)$ is the thermal conductivity of a grain layer with a moisture content;
λ_0 is the thermal conductivity of a layer with perfectly dry grain;
u is the moisture content in the grain of a layer;
c is the specific heat of the grain;
γ is the bulk density of a grain layer;
Fo_x is the Fourier number at a distance x;
c_0 is the specific heat of perfectly dry grain.

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